TRANSPORT AND STABILITY IMPLICATIONS FOR SHAPE AND ASPECT RATIO OF STEADY-STATE, HIGH-PERFORMANCE TOKAMAKS

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OUTLINE

I. Optimum tokamak study by Lin-Liu/Stambaugh

II. Comparison with similar study by Menard

III. Transport dependence on shape (A, κ)

I. WHAT IS THE OPTIMUM TOKAMAK?

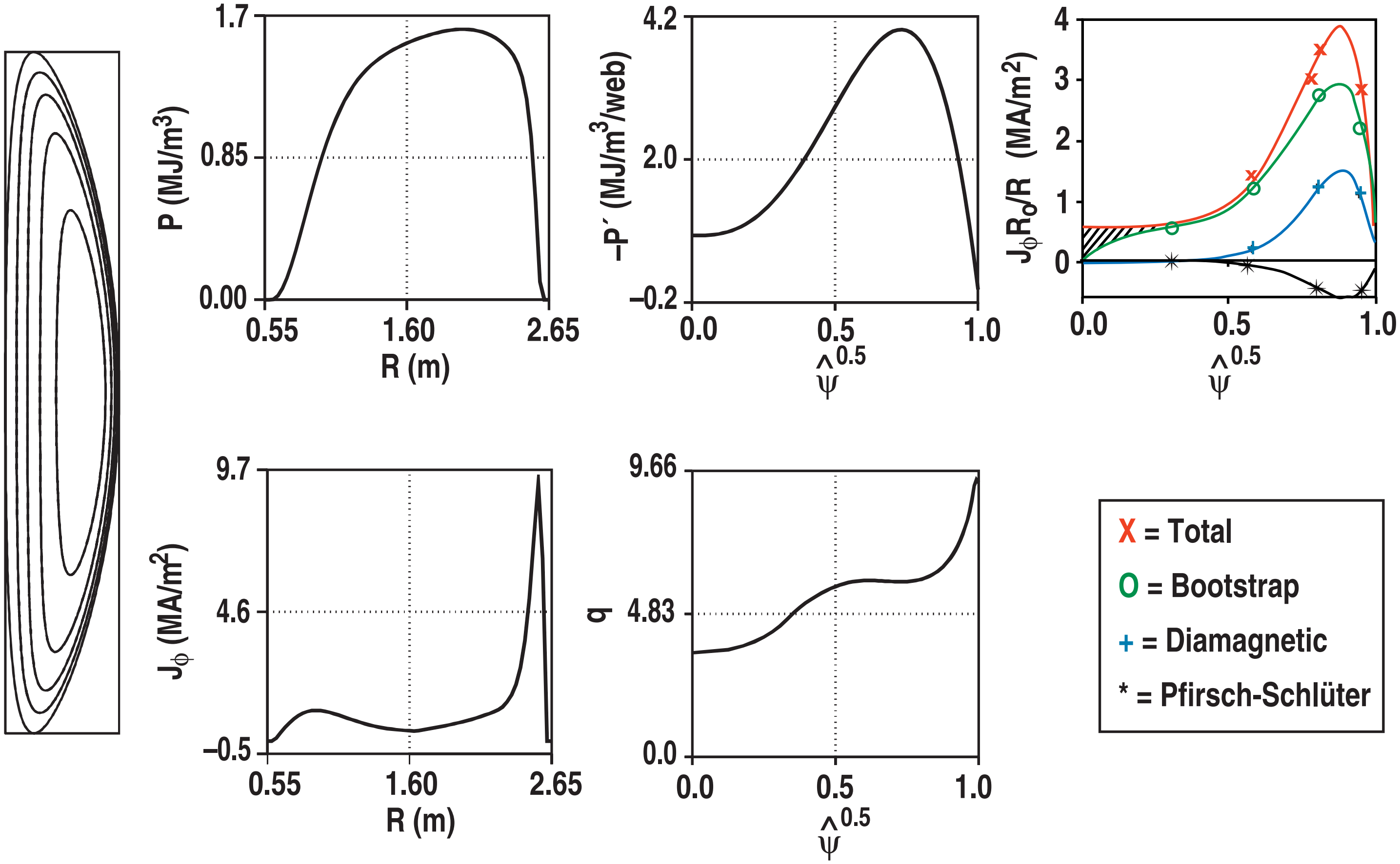
- Lin-Liu/Stambaugh constructed equilibria with
  - Bootstrap fraction of 99%, fully aligned
  - P´ = 0 at separatrix
  - Broad, nearly optimal, pressure profile
    - ★ Edge ITB?

- Ideal ballooning β limit found using BALOO
  - Bulk of plasma has second stability access
  - Ballooning limit occurs at a point near edge
  - Wall stabilization assumed for kinks

- Systematic shape study spanned
  - $1.5 \le \kappa \le 6.0$
  - $1.2 \le A \le 7.0$

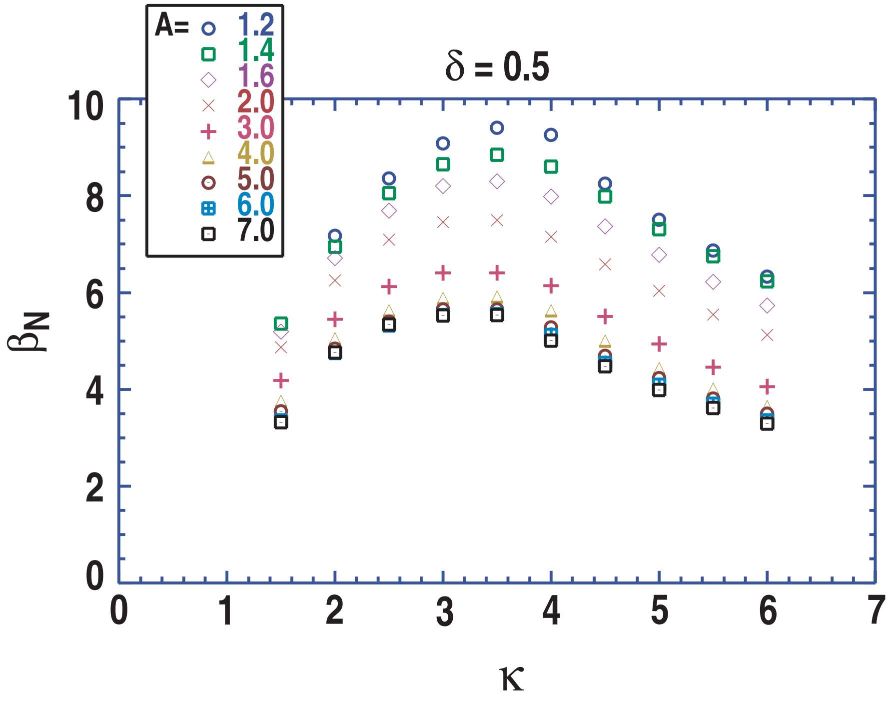
HIGH BETA, HIGH ELONGATION, HIGH BOOTSTRAP EQUILIBRIUM

- $A = 1.6$, $\kappa = 4.0$, $\delta = 0.5$, $\beta_T = 73\%$, $\beta_N = 8.0$, $\beta_P = 1.6$



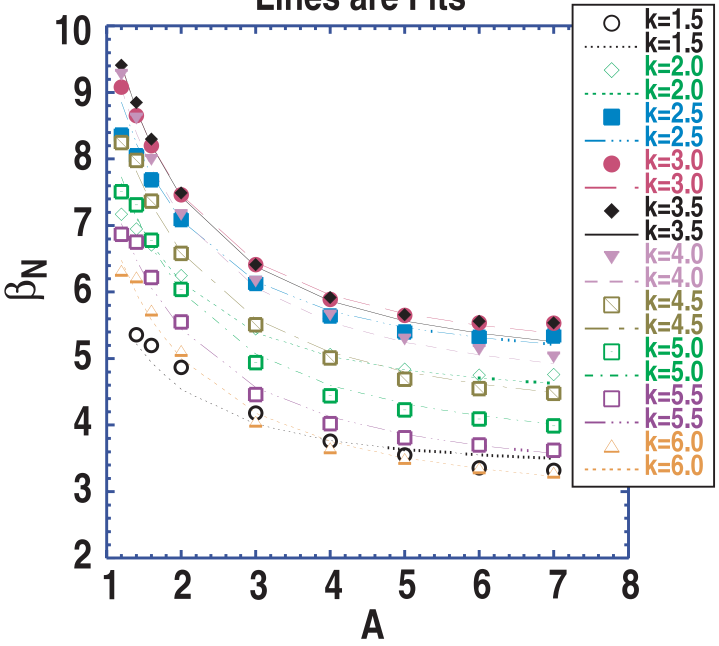
SYSTEMATIC STUDY OF β_N-LIMIT VERSUS R/a AND κ FOR f_{BS} = 0.99

β_N is optimal at κ = 3–4



β_N dependence close to $A^{-1/2}$

Symbols are Calculations
Lines are Fits



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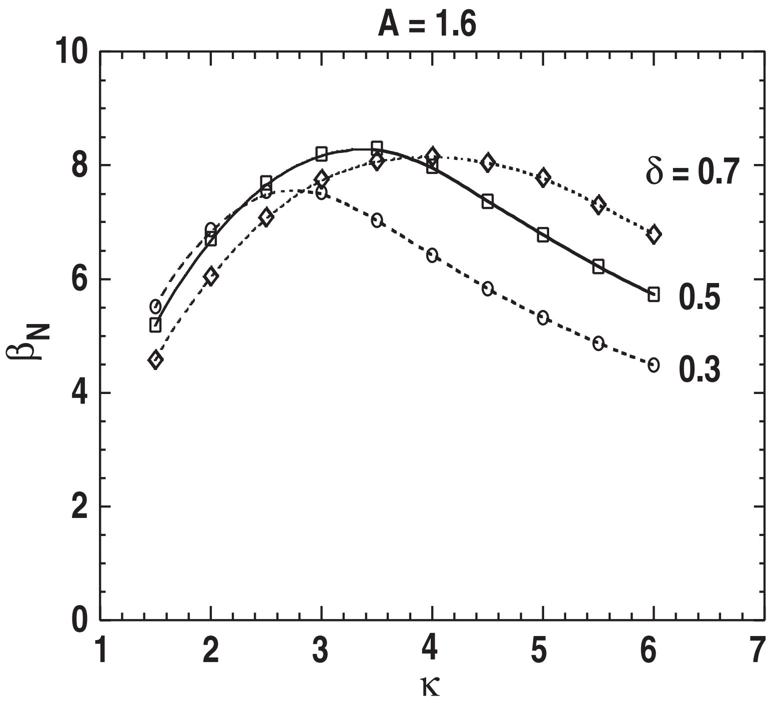
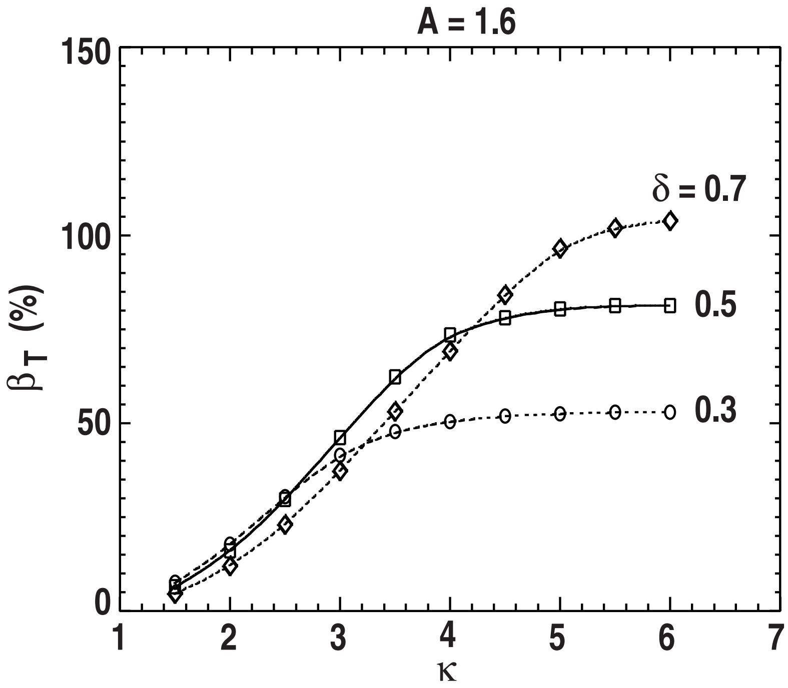
STRONG SHAPING (δ) IS NEEDED TO TAKE FULL ADVANTAGE OF HIGH ELONGATION

- Beta increases with δ for $\kappa \gtrsim 3$





KEY RESULTS OF LIN-LIU/STAMBAUGH STUDY

- Trade-off between fusion power and bootstrap current at a given normalized beta

$$\beta_T \, \beta_P = 25 \left(\frac{1+\kappa^2}{2}\right) \left(\frac{\beta_N}{100}\right)^2$$

$$f_{BS} = C_{BS} \, \beta_P / \sqrt{A}$$

$$P_F \propto \beta_T^2 \, B^4$$

- Shape dependence of ideal ballooning stable beta

$$\beta_N = 10 \left(b_0 + b_1\kappa + b_2\kappa^2 + b_3\kappa^3\right) \coth\left(\frac{d_o + d_1\kappa}{A^m}\right) \frac{1}{A^n}$$

b_0	–0.7748	d_0	1.8524
b_1	1.2869	d_1	0.2319
b_2	–0.2921	m	0.6163
b_3	0.0197	n	0.5523

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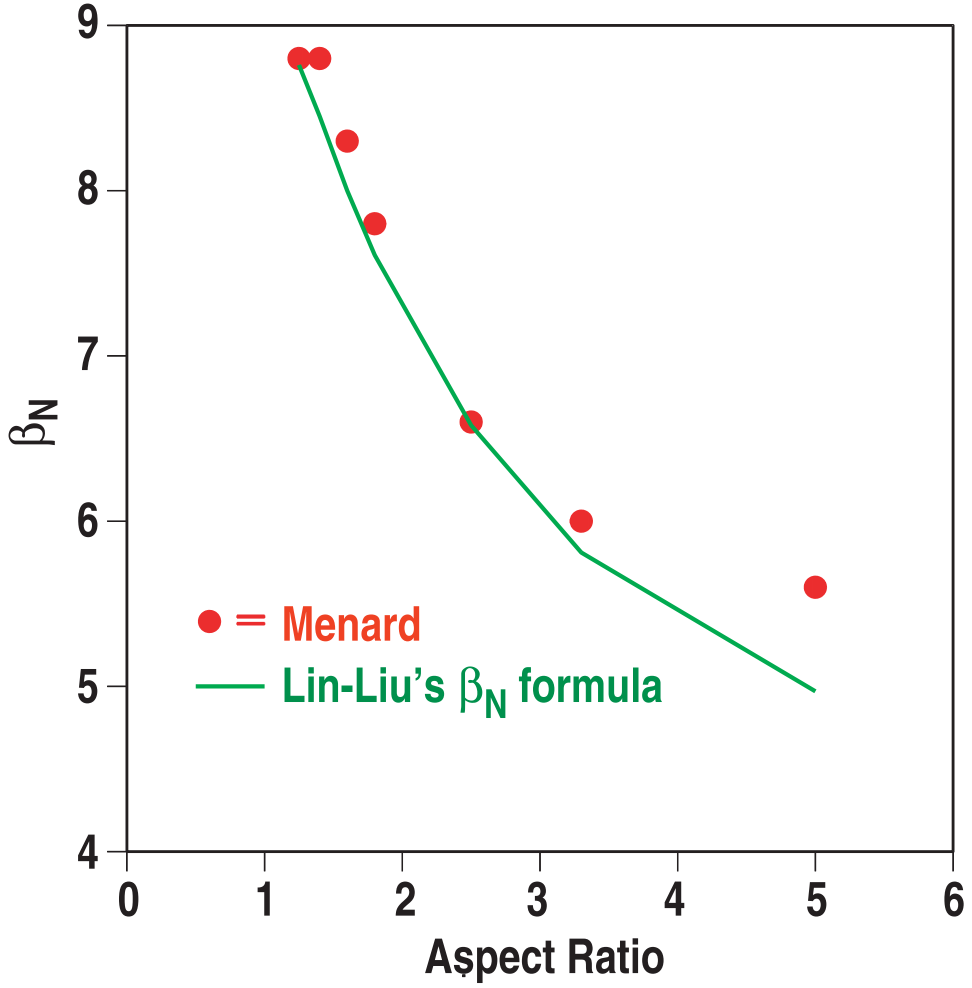
II. IDEAL WITH-WALL BALLOONING LIMIT FOR FULLY SELF-SUSTAINED EQUILIBRIA NEARLY SAME BETWEEN MENARD'S AND LIN-LIU'S STUDIES



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III. TRANSPORT DEPENDENCE ON SHAPE (A, κ)

- While empirical confinement scaling relations of the form

  IPB98(y,2) $\tau=0.0562\, I^{0.93}\, n^{0.41}\, B^{0.15}\, P^{-0.69}\, R^{1.97}\, m^{0.19}\, \kappa^{0.78}\, A^{-0.58}$

  EGB $\tau=0.028\, I^{0.83}\, n^{0.49}\, B^{0.07}\, P^{-0.55}\, R^{2.11}\, m^{0.14}\, \kappa^{0.75}\, A^{-0.3}$

  are fully predictive, the trade-offs between different parameters due to operational constraints are not readily apparent

  - ★ e.g., safety factor constraint relates I, κ, A, and R

- Casting confinement scaling relations in terms of dimensionless parameters allows the shape and aspect ratio dependences to be easily determined once the operational constraints are specified
  - Can choose kinetic plasma physics parameters like ρ_* and ν_*
  - Can choose MHD parameters like q and β_N

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DEPENDENCE OF TRANSPORT ON q AND κ

- Experiments on DIII–D resolved the ambiguity between the κ and q scalings of transport by comparing
  - q scan at fixed κ
  - κ scan at fixed q
  - κ scan at fixed I
- For H–mode plasmas, the change in confinement for the above three scans was explained by the unified scaling

$$B\tau \propto q_{95}^{-1.4\pm0.6}\ \kappa^{2.2\pm0.6}$$

- Note that the q and κ scalings of normalized confinement are different than the I_p and κ scalings of τ
  - ★ Converting dimensionless parameter scalings for H–mode plasmas on DIII–D to engineering parameter scalings gives

$$\tau \propto I_p^{0.76\pm0.14}\ \kappa^{0.65\pm0.16}$$

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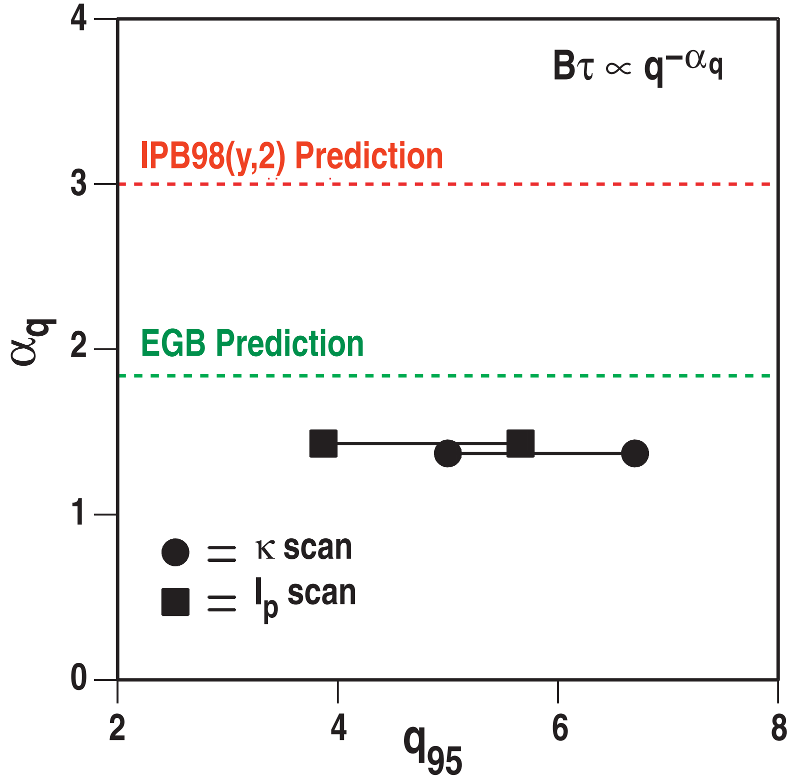
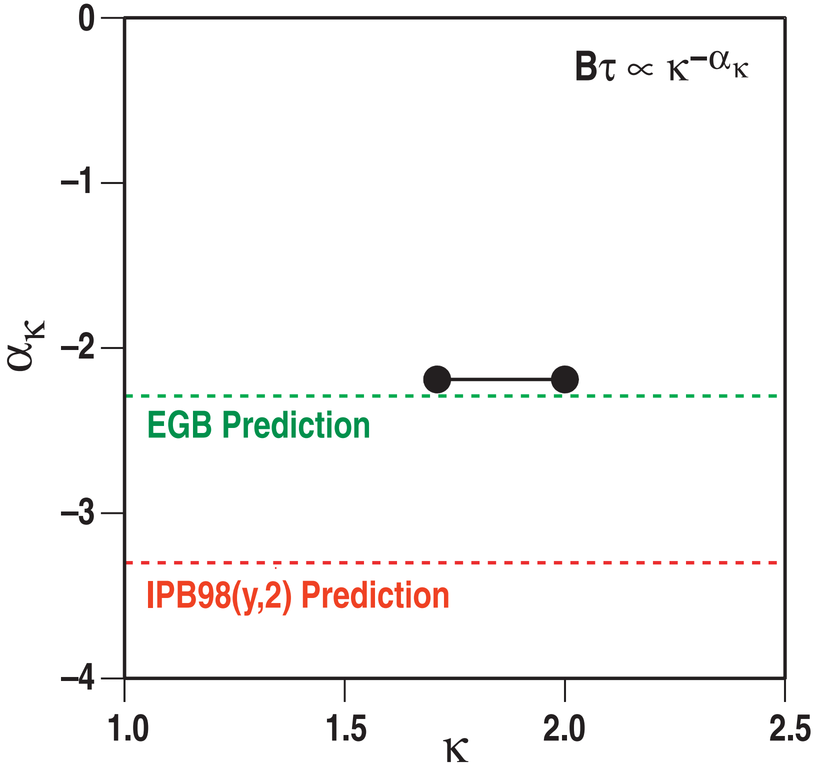
MEASURED q AND κ SCALINGS OF H–MODE TRANSPORT ARE WEAKER THAN PREDICTION FROM IPB98(y,2)

q scanned in two ways



κ scanned at fixed q



OPERATIONAL CONSTRAINTS ARE CRITICAL WHEN PROJECTING ASPECT RATIO SCALING OF TRANSPORT

- Aspect ratio affects many important dimensionless parameters
  - q, β_N, ν_*, f_{BS}, etc.
- Future experiments between DIII–D and NSTX/MAST will directly measure aspect ratio scaling of transport
- For steady-state, high-performance tokamaks, the aspect ratio scaling of confinement is more easily projected by substituting operational constraints for engineering parameters in scaling relations

$$\begin{pmatrix} I_p \\ B \\ n \\ P \end{pmatrix} \longleftrightarrow \begin{pmatrix} \rho_* \\ \beta_N \\ f_{BS} \\ f_{GR} \end{pmatrix}$$

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INCLUDING EFFECT OF ASPECT RATIO ON β_N, κ CHANGES R/a DEPENDENCE OF NORMALIZED CONFINEMENT

- Confinement scaling relations converted to dimensionless parameters

IPB98(y,2) $B\tau \propto \rho_*^{-2.7} A^{1.2} \kappa^{2.4} \beta_N^{1.2} f_{BS}^{-2.1} f_{GR}^{0.0}$

EGB $B\tau \propto \rho_*^{-3.3} A^{1.8} \kappa^{2.2} \beta_N^{2.1} f_{BS}^{-1.7} f_{GR}^{-0.6}$

- Include optimum tokamak scalings $\beta_N \propto A^{-1/2}$, $\kappa \propto A^{-1/2}$

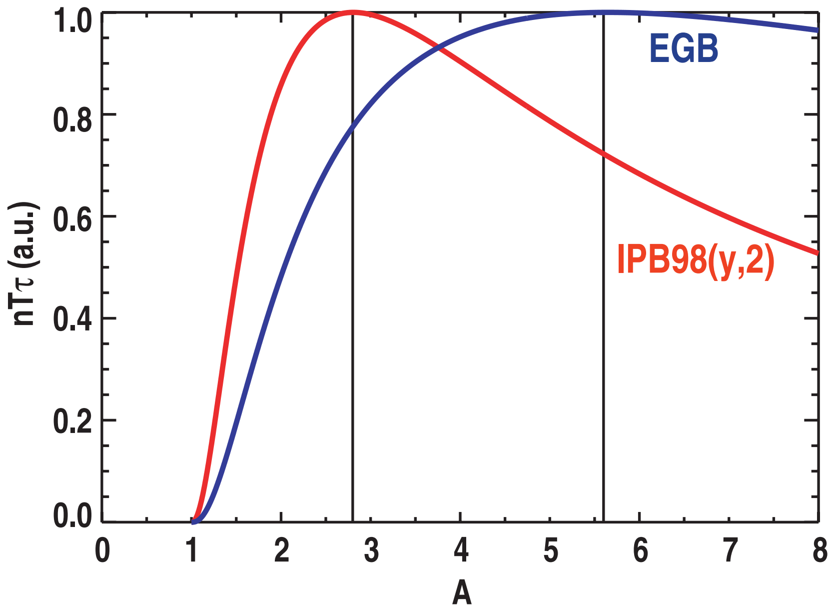
IPB98(y,2) $B\tau \propto \rho_*^{-2.7} A^{-0.6} f_{BS}^{-2.1} f_{GR}^{0.0}$

EGB $B\tau \propto \rho_*^{-3.3} A^{-0.3} f_{BS}^{-1.7} f_{GR}^{-0.6}$

FUSION GAIN OPTIMIZES FOR A = 2.2–3.0 FOR STEADY-STATE, HIGH-PERFORMANCE TOKAMAKS AT STABILITY LIMIT

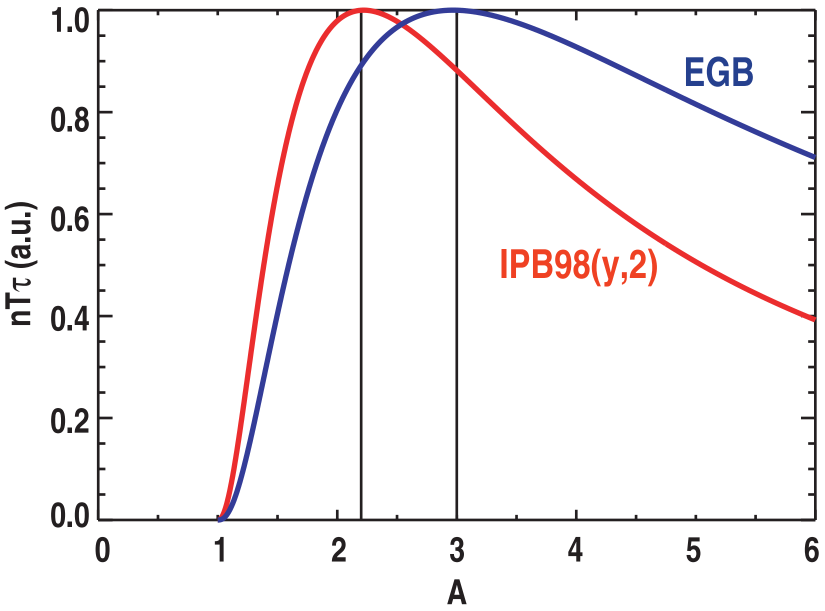
Fixed β_N, κ, f_{BS}, f_{GR} path

$\rho_*^{-1} \propto a^{1/2} B^{1/2}$



Fixed f_{BS}, f_{GR} path with β_N, $\kappa \propto A^{-1/2}$

$\rho_*^{-1} \propto a^{1/2} B^{1/2} A^{1/4}$



- Assume $B = B_c (1 - A^{-1})$ where B_c (= field at centerpost) is fixed

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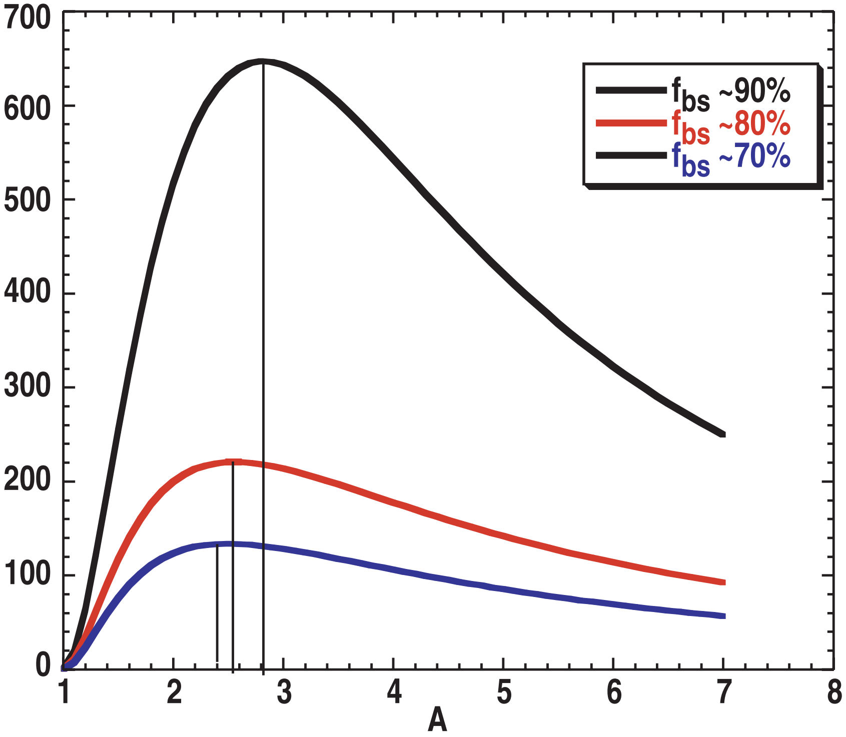
RECIRCULATING POWER FRACTION OPTIMIZES FOR A=2.4–2.8 AT STABILITY LIMIT

$$Q = \frac{P_F}{P_{CD}} = \frac{\gamma_{CD} P_F}{nIR(1 - f_{bs})} = \frac{\gamma_{CD}\, \beta_N^2\, \kappa B_c^2 \left(1 - \frac{1}{A}\right)^2 Ra^2}{f_{GR}\, R(1 - f_{bs})}$$

- $n = f_{GR} \dfrac{I}{\pi a^2}$
- B_C = field at centerpost (fixed maximum from stress)
- $f_{bs} = c_{bs}\, \beta_p / \sqrt{A} = \dfrac{c_{bs}}{20} \sqrt{A}\ \ q_{cyl}\ \ \beta_N$
- Express $\beta_N(A)$ as $\beta_{N_O} A^{-\alpha}$ and κ as $\kappa_O A^{-\phi}$
- Optimize the function $$\frac{A^{-2\alpha}\left(1 - \frac{1}{A}\right)^2 A^{-\phi}}{\left(1 - \frac{c_{bs}}{20}\ q_{cyl}\ \beta_{N_O}\ A^{1/2-\alpha}\right)}$$

for $\alpha = 1/2$; $\phi = 1/2$ $A_{max} = 2.3$



KEY RESULTS OF TRANSPORT DEPENDENCE ON SHAPE (A, κ)

- Transport dependence on elongation and safety factor are weaker than IPB98(y,2) relation but close to EGB relation
- Transport dependence on aspect ratio is more apparent when the operational constraints (β_N, κ, f_{BS}, f_{GR}) are directly incorporated into the confinement scaling relation
- For “optimum tokamak”, fusion gain is optimized between aspect ratio of 2.2 and 3.0 (depending upon which confinement scaling relation is used)
- If stability limit and elongation are assumed independent of aspect ratio, then fusion gain optimizes at higher R/a